№478

$f(x) = x(x-2), \qquad F(x) = e^x$

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

$$U\Big|_{t_0=0} = f(x); \qquad \frac{\partial u}{\partial t}\Big|_{t_0=0} = F(x)$$

$$U = \frac{f(x-at)+f(x+at)}{2} + \frac{1}{2a}\int_{x-at}^{x+at} F(z)dz = \frac{(x-at)(x-at-2)+(x+at)(x+at-2)}{2} + \frac{1}{2a}\int_{x-at}^{x+at} e^z dz =$$

$$= \frac{x^2 - xat - 2x - atx + a^2t^2 + 2at + x^2 + xat - 2x + atx + a^2t^2 - 2at}{2} + \frac{1}{2a}\left(e^{x+at} - e^{x-at}\right) =$$

$$= \frac{2x^2 - 4x + 2a^2t^2}{2} + \frac{1}{2a}\left(e^{x+at} - e^{x-at}\right) = x^2 - 2x + a^2t^2 + \frac{1}{2a}e^{x+at} - e^{x-at}.$$

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№488

$$w = e^{-iz^{2e}} \qquad z_0 = \frac{\sqrt{\pi}i}{2}$$

$$w = e^{-iz^2} = e^{i(x+y_i)^2} = e^{i(x^2+2xyi-y^2)} = e^{x^2 i 2xy+iy^2} = e^{-2xy+(x^2-y^2)i} = e^{-2xy} * e^{(x^2+y^2)i} =$$

$$= e^{-2xy}(\cos(x^2-y^2) + i\sin(x^2-y^2)) = e^{-2xy}\cos(x^2-y^2) + ie^{-2xy}\sin(x^2-y^2)$$

$$u(x;y) = e^{-2xy}\cos(x^2-y^2)$$

$$v(x;y) = e^{-2xy}\sin(x^2-y^2)e$$

$$\frac{\partial u}{\partial x} = (e^{-2xy}\cos(x^2-y^2))'_x = (e^{-2xy})'_x\cos(x^2-y^2) + e^{-2xy}(\cos(x^2-y^2))'_x =$$

$$= e^{-2xy}(-2xy)'_x(\cos(x^2-y^2)) + e^{-2xy}(\sin(x^2-y^2))(x^2-y^2)'_x = -2ye^{-2xy}\cos(x^2-y^2) -$$

$$-2xe^{-2xy}\sin(x^2-y^2)$$

$$\frac{\partial v}{\partial y}(e^{-2xy}\sin(x^2-y^2))'_y = (e^{-2xy})'_y\sin(x^2-y^2) + e^{-2xy}(\sin(x^2-y^2))'_y =$$

$$= e^{-2xy}(-2xy)'_y\sin(x^2-y^2) + e^{-2xy}\cos(x^2-y^2)(x^2-y^2)'_y = -2xe^{-2xy}\sin(x^2-y^2) +$$

$$+ e^{-2xy}\cos(x^2-y^2)(-2y) = -2xe^{-2xy}\sin(x^2-y^2) - 2ye^{-2xy}\cos(x^2-y^2)$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = -2xe^{-2xy}\sin(x^2-y^2) - 2ye^{-2xy}\cos(x^2-y^2);$$

$$\frac{\partial v}{\partial x} = (e^{-2xy}\sin(x^2-y^2))'_x = (e^{-2xy})'\sin(x^2-y^2) + e^{-2xy}(\sin(x^2-y^2))'_x =$$

$$= -2ye^{-2xy}\sin(x^2-y^2) + e^{-2xy}\cos(x^2-y^2)(x^2-y^2)'_x = -2ye^{-2xy}\sin(x^2-y^2) + 2xe^{-2xy}\cos(x^2-y^2)$$

$$\frac{\partial u}{\partial y} = (e^{-2xy}\cos(x^2-y^2))'_y = (e^{-2xy})'_y(x^2-y^2) + e^{-2xy}(\cos(x^2-y^2))'_y =$$

$$= e^{-2xy}(-2xy)'_y\cos(x^2-y^2) + e^{-2xy}(-\sin(x^2-y^2))(x^2-y^2)'_y = -2xe^{-2xy}\cos(x^2-y^2) -$$

$$- e^{-2xy}\sin(x^2-y^2)(-2y) = -2xe^{-2xy}\cos(x^2-y^2) + 2e^{-2xy}\sin(x^2-y^2) =$$

$$= -(e^{-2xy}\sin(x^2-y^2) + e^{-2xy}\cos(x^2-y^2))$$

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} = -2e^{-2xy}\sin(x^2-y^2) + 2xe^{-2xy}\cos(x^2-y^2)$$

$$z' = \left.\frac{\partial u}{\partial x}\right|_{z_0} + \left.\frac{\partial v}{\partial y}\right|_{z_0}$$

$$\frac{\partial u}{\partial x} = -2xe^{-2xy}\sin(x^2-y^2) - 2ye^{-2xy}\cos(x^2-y^2)\Big|_{\frac{\sqrt{\pi}i}{2}} = -2*0*e^{-2*0*\frac{\sqrt{\pi}}{2}}\sin\left(0^2\left(\frac{\sqrt{\pi}}{2}\right)^2\right) -$$

$$-2\frac{\sqrt{\pi}}{2}e^{-2*0*\frac{\sqrt{\pi}}{2}}\cos\left(0^2\left(\frac{\sqrt{\pi}}{2}\right)^2\right) = 0 - \sqrt{\pi}e\cos\left(-\frac{\pi}{4}\right) = -\sqrt{\pi}\frac{\sqrt{2}}{2} = -\frac{\sqrt{\pi}\sqrt{2}}{2} = -\frac{\sqrt{\pi}}{\sqrt{2}}$$

$$\left.\frac{\partial v}{\partial x}\right|_{\frac{\sqrt{\pi}i}{2}} = -2ye^{-2xy}\sin(x^2-y^2) + 2xe^{-2xy}\cos(x^2-y^2)\Big|_{\frac{\sqrt{\pi}i}{2}} = 2\frac{\sqrt{\pi}}{2}e^{-2*0*\frac{\sqrt{\pi}i}{2}}\sin\left(0^2\left(\frac{\sqrt{\pi}}{2}\right)^2\right) +$$

$$+2*0e^{-2*0\frac{\sqrt{\pi}i}{2}}\cos\left(0^2 - \left(\frac{\sqrt{\pi}}{2}\right)^2\right) = -\sqrt{\pi}e^0\sin\frac{\pi}{4} + 0 = -\sqrt{\pi}\frac{\sqrt{2}}{2} = -\frac{\sqrt{\pi}}{\sqrt{2}} =$$

$$z' = -\frac{\sqrt{\pi}}{\sqrt{2}} - \frac{\sqrt{\pi}}{\sqrt{2}} i = -\sqrt{\frac{\pi}{2}}(1+i).$$

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№498

$f(z) = e^{\frac{1}{1-z}}; \qquad z_0 = 1$

Воспользуемся разложением:

$$e^z = 1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots.$$

Получаем:

$$f(z) = e^{\frac{1}{1-z}} = 1 + \frac{1}{1-z} + \frac{1}{(1-z)^2 2!} + \frac{1}{(1-z)^3 3!} + \dots$$

Радиус сходимости ряда найдем по признаку Даламбера:

$$\rho = \lim_{n\to\infty} \frac{1}{n!}(n+1)! = \lim_{n\to\infty}(n+1) = \infty$$

Следовательно ряд сходится при:

$$-\infty < |1-z| < +\infty.$$

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№508

$x''-4x=t-1; \qquad x(0)=0, \qquad x'(0)=0$

Переходим к изображениям:

$$p^2\bar{x}-px(0)-x'(0)-4\bar{x}=\frac{1}{p^2}-\frac{1}{p}$$

$$p^2\bar{x}-4\bar{x}=\frac{1-p}{p^2}$$

$$\bar{x}(p^2-4)=\frac{1-p}{p^2}$$

$$\bar{x}=\frac{1-p}{p^2(p^2-4)}$$

Разложим эту рациональную дробь на простейшие дроби:

$$\frac{1-p}{p^2(p^2-4)}=\frac{A}{p}+\frac{B}{p^2}+\frac{C}{p-2}+\frac{D}{p+2}=\frac{Ap(p^2-4)+B(p^2-4)+Cp^2(p+2)+Dp^2(p-2)}{p^2(p^2-4)}$$

$$\begin{array}{l|l} p^3 & A+C+D=0 \\ p^2 & B+2C-2D=0 \\ p^1 & -4A=-1 \\ p^0 & -4B=1 \end{array} \Rightarrow \quad A=\frac{1}{4}; \quad B=-\frac{1}{4}$$

$$C=-\frac{1}{4}-D$$

$$-\frac{1}{4}-\frac{1}{2}-4D=0\Rightarrow$$

$$D=-\frac{3}{16}; \qquad C=-\frac{1}{4}+\frac{3}{16}=-\frac{1}{16}$$

$$\bar{x}=\frac{1}{p}\frac{1}{4}-\frac{1}{4}\frac{1}{p^2}-\frac{1}{16}\frac{1}{p-2}-\frac{3}{16(p+2)}, \quad \text{откуда} \quad x=\frac{1}{4}-\frac{1}{4}t-\frac{1}{16}e^{2t}-\frac{3}{16}e^{-2t}$$

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№518

$$\begin{cases} x' = y - z \\ y' = x + y; \\ z' = x + z \end{cases} \qquad x(0) = 1 \qquad y(0) = 2 \qquad z(0) = 3$$

Перейдя к изображениям, имеем:

$$\begin{cases} p\bar{x} - x(0) = \bar{y} - \bar{z} \\ p\bar{y} - y(0) = \bar{x} + \bar{y} \\ p\bar{z} - z(0) = \bar{x} + \bar{z} \end{cases} \text{ или } \begin{cases} p\bar{x} = \bar{y} - \bar{z} + 1 \\ p\bar{y} = \bar{x} + \bar{y} + 2 \Rightarrow \\ p\bar{z} = \bar{x} + \bar{z} + 3 \end{cases} \quad \bar{y}(p-1) = \bar{x} + 2 \quad \bar{y} = \frac{\bar{x}+2}{p-1} \qquad \bar{z}(p-1) = \bar{x} + 3 \quad \bar{z} = \frac{\bar{x}+3}{p-1}$$

$$p\bar{x} = \frac{\bar{x}+2}{p-1} - \frac{\bar{x}+3}{p-1} + 1$$

$$p\bar{x} = \frac{\bar{x} + 2 - \bar{x} - 3 + p - 1}{p-1} = \frac{p-2}{p-1}$$

$$\bar{x} = \frac{p-2}{p(p-1)}$$

$$\frac{p-2}{p(p-1)} = \frac{A}{p} + \frac{B}{p-1} = \frac{A(p-1) + Bp}{p(p-1)}$$

$$\begin{array}{c|l} p' & A + B = 1 \\ p^0 & -A = -2 \end{array} \Rightarrow A = 2; \qquad B = 1 - 2 = -1$$

$$\bar{x} = \frac{2}{p} - \frac{1}{p-1}, \text{ откуда } x = 2 - e^t$$

$$\bar{y} = \frac{\frac{p-2}{p(p-1)} + 2}{p-1} = \frac{p - 2 + 2p(p-1)}{p(p-1)^2} = \frac{2p^2 - p - 2}{p(p-1)^2}$$

$$\frac{2p^2 - p - 2}{p(p-1)^2} = \frac{A}{p} + \frac{B}{p-1} + \frac{C}{(p-1)^2} = \frac{A(p^2 - 2p + 1) + Bp(p-1) + Cp}{p(p-1)^2}$$

$$\begin{array}{c|c} p^2 & A + B = 2 \\ p^1 & -2A - B + C = -1 \Rightarrow \\ p^0 & A = -2 \end{array} \quad \begin{array}{c} B = 4 \\ 4 - 4 + C = -1 \Rightarrow C = -1 \end{array}$$

$$\bar{y} = -\frac{2}{p} + \frac{4}{p-1} - \frac{1}{(p-1)^2}, \text{ откуда } y = -2 + 4e^t - te^t$$

$$\bar{z} = \frac{\frac{p-2}{p(p-1)} + 3}{p-1} = \frac{p - 2 + 3p(p-1)}{p(p-1)^2} = \frac{3p^2 - 2p - 2}{p(p-1)^2}$$

$$\frac{3p^2 - 2p - 2}{p(p-1)^2} = \frac{A}{p} + \frac{B}{p-1} + \frac{C}{(p-1)^2}$$

$$\begin{array}{c|c} p^2 & A + B = 3 \\ p' & -2A - B + C = -2 \Rightarrow \\ p^0 & A = -2 \end{array} \quad \begin{array}{c} B = 5 \\ 4 - 5 + C = -2 \\ C = -1 \end{array}$$

$z = -\frac{2}{p} + \frac{5}{p-1} + \frac{1}{(p-1)^2}$, откуда

$\mathrm{z} = -2 + 5\mathrm{e}^{\mathrm{t}} - te^{t}$

Ответ:

$\mathrm{x} = 2 - \mathrm{e}^{\mathrm{t}}$

$y = -2 + 4\mathrm{e}^{\mathrm{t}} - t\mathrm{e}^{\mathrm{t}}$

$z = -2 + 5\mathrm{e}^{\mathrm{t}} - t\mathrm{e}^{\mathrm{t}}$