№286

$$а)\int \frac{\sin dx}{\sqrt[3]{\cos^2 x}} = -\int \frac{d\cos x}{\sqrt[3]{\cos^2 x}} = -\int \cos^{-2/3} xdcodx = -\frac{\cos^{\frac{1}{3}} x}{\frac{1}{3}} + C = -3\sqrt[3]{\cos x} + C.$$

$$\text{Проверка}: \left(-3\sqrt[3]{\cos x} + C\right)' = -3 * \frac{1}{3}(\cos x)^{-\frac{2}{3}}(-\sin x) = \frac{\sin x}{\sqrt[3]{\cos^2 x}}$$

$$б)\int x \arcsin\frac{1}{x}dx = \left[\begin{array}{c} u = \arcsin\frac{1}{x},\ du = -\frac{1}{\sqrt{1-\frac{1}{x^2}}} * \frac{1}{x^2}dx \\ dV = xdx;\quad V = \frac{x^2}{2} \end{array}\right] =$$

$$= \frac{x^2}{2}\arcsin\frac{1}{x} + \int \frac{x^2}{2} * \frac{1}{\sqrt{1-\frac{1}{x^2}}} * \frac{1}{x^2}dx = \frac{x^2}{2}\arcsin\frac{1}{x} + \frac{1}{2}\int \frac{dx}{\sqrt{1-\frac{1}{x^2}}} = \frac{x^2}{2}\arcsin\frac{1}{x} + \frac{1}{4}\int \frac{d(x^2-1)}{\sqrt{x^2-1}} =$$

$$= \frac{x^2}{2}\arcsin\frac{1}{x} + \frac{1}{4}2\sqrt{x^2-1} + C = \frac{x^2}{2}\arcsin\frac{1}{x} + \frac{1}{2}\sqrt{x^2-1} + C.$$

$$\text{Проверка}: \left(\frac{x^2}{2}\arcsin\frac{1}{x} + \frac{1}{2}\sqrt{x^2-1} + C\right)' = \frac{2x}{2}\arcsin\frac{1}{x} + \frac{x^2}{2} * \frac{1}{\sqrt{1-\frac{1}{x}}}\left(-\frac{1}{x^2}\right) + \frac{1}{2} * \frac{1}{2}\frac{1}{\sqrt{x^2-1}} * 2x =$$

$$= x\arcsin\frac{1}{x} - \frac{1}{2}\frac{x}{\sqrt{x^2-1}} + \frac{1}{2}\frac{x}{\sqrt{x^2-1}} = x\arcsin\frac{1}{x}$$

$$в)\int \frac{(x+3)dx}{x^3+x^2-2x} = I$$

$$\frac{x+3}{x^3+x^2-2x} = \frac{x+3}{x(x^2+x-2)} = \frac{x+3}{x(x+2)(x-1)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-1} =$$

$$= \frac{A(x^2+x-2) + B(x^2-x) + C(x^2+2x)}{x(x+2)(x-1)} = \frac{x^2(A+B+C) + x(A-B+2C) + (-2A)}{x(x+2)(x-1)};$$

$$\begin{array}{l|l} x^2 & A+B+C=0 \\ x^1 & -A+2A-B+2C=1 \Rightarrow \\ x^0 & -2A=3 \end{array} \qquad \begin{array}{c} A = -\tfrac{3}{2} \\ B = -C + \tfrac{3}{2} \\ -\tfrac{3}{2} + C - \tfrac{3}{2} + 2C = 1 \\ 3C = 1 + \tfrac{6}{2} = \tfrac{8}{2} = 4 \\ C = \tfrac{4}{3} \Rightarrow B = -\tfrac{4}{3} + \tfrac{3}{2} = \tfrac{1}{6} \end{array}$$

$$\begin{array}{l|l} x^2 & A+B+C=0 \\ x^1 & A-B+2C=1 \\ x^0 & -2A=3 \quad A = -\tfrac{3}{2} \end{array} \qquad \begin{array}{c} 2A+3C=1 \\ 2\left(-\tfrac{3}{2}\right)+3C=1 \\ 3C=4 \quad C=\tfrac{4}{3} \\ B=\tfrac{1}{6} \end{array}$$

$$I = -\frac{3}{2}\int\frac{dx}{x} + \frac{1}{6}\int\frac{dx}{x+2} + \frac{4}{3}\int\frac{2x}{x-1} = -\frac{3}{2}\ln x + \frac{1}{6}\ln(x+2) + \frac{4}{3}\ln(x-1) + C$$

$$г)\int \frac{\left(\sqrt[4]{x}+1\right)}{\left(\sqrt{x}+4\right)\sqrt[4]{x^3}} = \left[\begin{array}{c} \sqrt[4]{x} \\ x = t^4 \\ dx = 4t^3dt \end{array}\right] = \int \frac{(t+1)}{(t^2+4)t^3} * 4t^3dt = 4\int\frac{t+1}{t^2+4}dt = 4\int\frac{t}{t^2+4}dt + 4\int\frac{dt}{t^2+4} =$$

$$= 2\int\frac{d(t\ \ +4)}{t^2+4} + 2\int\frac{d\left(\frac{t}{2}\right)}{\left(\frac{t}{2}\right)^2+1} = 2\ln(t^2+4) + 2arctg\left(\frac{t}{2}\right) + C = 2\ln\left(\sqrt{x}+4\right) + 2arctg\left(\frac{\sqrt[4]{x}}{2}\right) + C.$$

№296

$$\int_2^{12} \sqrt{x^2+4}dx$$

$$a = 2 \qquad b = 12 \qquad h = \frac{b - a}{n} = \frac{12-2}{10} = 1;$$

Нужно определить значения подынтегральной функции для следующих значений аргумента $(h=1)$:

$x_0 = 2;\ x_1 = 3;\ x_2 = 4;\ x_3 = 5;\ x_4 = 6;\ x_5 = 7;\ x_6 = 8;\ x_7 = 9;\ x_8 = 10;\ x_9 = 11;\ x_{10} = 12.$

Находим:

$y_0 = \sqrt{8} = 2{,}828;\ y_1 = 3{,}606;\ y_2 = 4{,}472;\ y_3 = 5{,}385;\ y_4 = 6{,}325;\ y_5 = 7{,}28;\ y_6 = 8{,}246;\ y_7 = 9{,}22;$
$y_8 = 10{,}198;\ y_9 = 11{,}18;\ y_{10} = 12{,}166.$

$$\int_2^{12} \sqrt{x^2+4}dx = \frac{h}{3}\left[y_0 + y_{10} + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8)\right] =$$

$$= \frac{1}{3}\left[2{,}828 + 12{,}166 + 4(3{,}606 + 5{,}385 + 7{,}28 + 9{,}22 + 11{,}18) + 2(4{,}472 + 6{,}325 + 8{,}246 + 10{,}198)\right] =$$

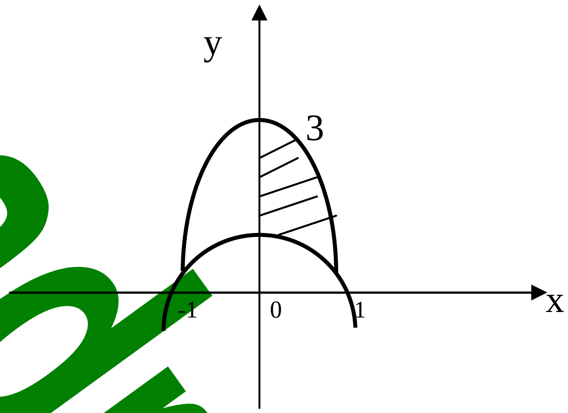
$$= \frac{1}{3}\left[14{,}994 + 146{,}684 + 58{,}482\right] = 73{,}387.$$

№306

$$\int_{-3}^{2} \frac{dx}{(x+3)^2} = \lim_{a\to -3} \int_a^2 \frac{dx}{(x+3)^2} = \lim_{a\to -3} -\frac{1}{x+3}\bigg|_a^2 = \lim_{a\to -3}\left(-\frac{1}{5} + \frac{1}{a+3}\right) = \infty \Rightarrow$$ данный несобственный инеграл расходится.

№316

$y = 3\sqrt{1-x^2}$ – полуэлипс; $x = \sqrt{1-y}$ – парабола, ось ОУ.



$$Vx = \pi\int_a^b (y_2^2 - y_1^2)dx$$

$$V = \pi\int_0^1 (9(1-x^2) - (1-x^2)^2)dx = \pi\int_0^1 (9 - 9x^2 - 1 + 2x^2 - x^4)dx = \pi\int_0^1 (8 - 7x^2 - x^4)dx =$$

$$= \pi\left(8x - \frac{7x^3}{3} - \frac{x^5}{5}\right)\Bigg|_0^1 = \pi\left(8 - \frac{7}{3} - \frac{1}{5}\right) = \pi\frac{120 - 35 - 3}{15} = \frac{82}{15}\pi(куб.ед.).$$