№375

$$x^4 = a^2(3 * x^2 - y^2) \quad \begin{cases} x = p\cos\phi \\ y = p\sin\phi \end{cases}$$

$$p^4\cos^4\varphi = a^2\left(3p^2\cos^2\varphi - p^2\sin^2\varphi\right)$$

$$p^4\cos^4\varphi = a^2p^2\left(3\cos^2\varphi - \sin\varphi\right)$$

$$p^2 = \frac{a^2}{\cos^4\varphi}(3\cos^2\varphi - \sin^2\varphi)$$

$$p = \frac{a}{\cos^2\varphi}\sqrt{3\cos^2\varphi - \sin^2\varphi} = \frac{a}{\cos^2\varphi}\sqrt{3\frac{1+\cos 2\varphi}{2} - \frac{1-\cos 2\varphi}{2}} = \frac{a}{\cos^2\varphi}\sqrt{1+2\cos 2\varphi}.$$

$$1 + 2\cos 2\varphi \ge 0$$

$$\cos 2\varphi \ge -\tfrac{1}{2}$$

$$-\tfrac{2\pi}{3} \le 2\varphi \le \tfrac{2\pi}{3}$$

$$\tfrac{\pi}{3} \le \varphi \le \tfrac{\pi}{3}$$

$$\varphi \ne 0$$

Т.к. кривая симметрична относительно осей координат, то достаточно рассмотреть $\varphi \in [0;\pi/2]$ и умножить результат на "4" - так как $2^2 \le 0$, то

$$\begin{cases} \phi \in [0;\pi/2] \\ 3\cos^2\phi - \sin^2\phi \ge 0 \end{cases} \Rightarrow \begin{cases} \phi \in [0;\pi/2] \\ \operatorname{tg}\varphi \le \sqrt{3} \end{cases} \Rightarrow \phi \in \left[0;\pi/3\right]$$

Итак:

$$\frac{S}{4}\iint dxdy = \int_0^{\pi/3} d\phi \int_0^{r(\phi)} dr * r = \frac{1}{2}\int_0^{\pi/3} r^2\Big|_0^{r(\phi)} d\phi = \frac{a^2}{2}\int_0^{\pi/3}\frac{3\cos^2\phi - \sin^2\phi}{\cos^4\phi}d\phi = \frac{3a^2}{2}\int_0^{\pi/3}\frac{d\phi}{\cos^2\phi} - \frac{a^2}{6}\int_0^{\pi/3}\sin\phi *$$

$$* d\left(\frac{1}{\cos^3\phi}\right) = \frac{3a^2}{2}\int_0^{\pi/3}\frac{d\phi}{\cos 2\phi} - \frac{a^2\sin\phi}{6\cos^3\phi}\Bigg|_0^{\pi/3} + \frac{a^2}{6}\int_0^{\pi/3}\frac{d\phi}{\cos^2\phi} = \frac{5a^2}{3}\int_0^{\pi/3}\frac{d\phi}{\cos^2\phi} = \frac{a^2\sqrt{3}/2}{6\left(1/2\right)^3} =$$

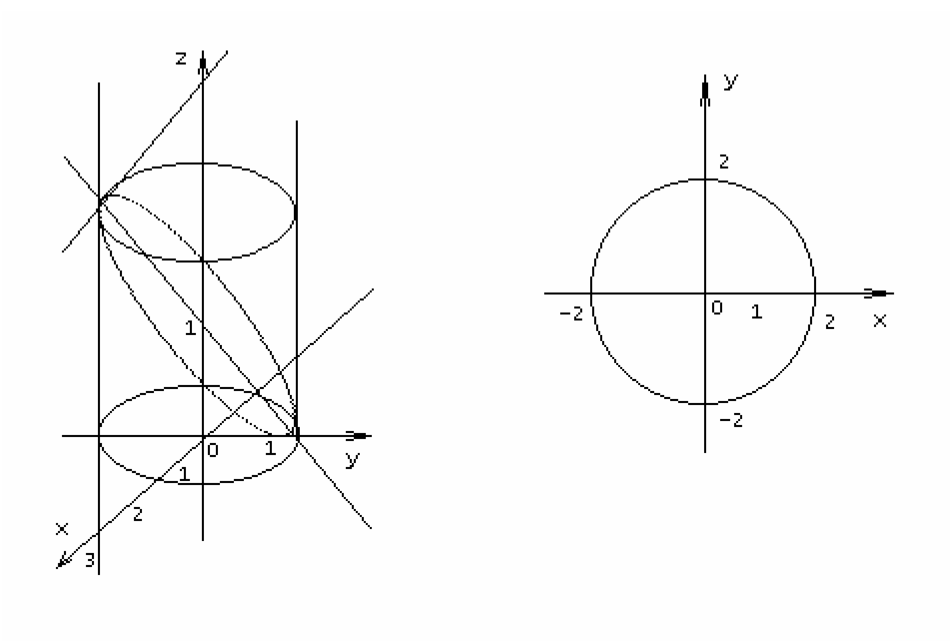
$$= \frac{5a}{3}tg\phi\Big|_0^{\pi/3} - \frac{2a^2\sqrt{3}}{3} = \frac{5a^2}{3}\sqrt{3} - \frac{2a^2}{3}\sqrt{3} = a^2\sqrt{3} \Longrightarrow S = 4\sqrt{3}a^2.$$

№385

$z=0,\quad y+z=2,\quad x^2+y^2=4$

Объём тела найдём по формуле:

$$V=\iiint_T dxdydz$$



$$V=\int_{-2}^{2}dx\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}dy\int_{0}^{2-y}dz=\int_{-2}^{2}dx\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}(2-y)dy=\int_{-2}^{2}\left(2y-\frac{y^2}{2}\right)\Bigg|_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}dx=$$

$$\int_{-2}^{2}\left(2\sqrt{4-x^2}-\frac{4-x^2}{2}+2\sqrt{4-x^2}+\frac{4-x^2}{2}\right)dx=4\int_{-2}^{2}\sqrt{4-x^2}dx=8*2\int_{-2}^{2}\sqrt{1-\left(\frac{x}{2}\right)^2}d\left(\frac{x}{2}\right)=$$

$$=\left[\begin{array}{l}\frac{x}{2}=\sin t\\ d\frac{x}{2}=\cos tdt\end{array}\right]=16\int_{-\pi/2}^{\pi/2}\frac{1+\cos 2t}{2}dt=(8t+4\sin 2t)\Bigg|_{-\pi/2}^{\pi/2}=8\left(\frac{\pi}{2}+\frac{\pi}{2}\right)=8\pi(\text{куб.ед.})$$

№395

$$\int_L (x^2 y - 3x)dx + (y^2 x + 2y)dy.$$

$$\begin{cases} x = 3\cos t \\ y = 2\sin t \end{cases} \Rightarrow \begin{cases} dx = -3\sin t dt \\ dy = 2\cos t dt \end{cases}$$

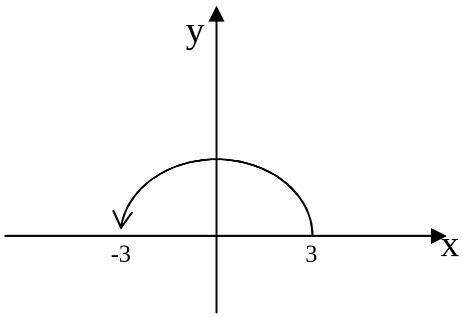
Тогда:

$$\int_L (x^2 y - 3x)dx + (y^2 x + 2y)dy =$$

$$= \int_0^{\pi} [(9\cos^2 t * 2\sin t * 9\cos t) * (-3\sin t) + (4\sin^2 t * 3\cos t + 4\sin t) * 2\cos t]dt =$$

$$= \int_0^{\pi} (35\sin t\cos t - 30\cos^2 t\sin^2 t)dt = -\frac{15}{2}\int_0^{\pi}\frac{1-\cos^4 t}{2}dt - \frac{35}{4}\cos 2t\Big|_0^{\pi} =$$

$$= -\frac{15}{4}t\Big|_0^{\pi} + \frac{15}{16}\sin 4t\Big|_0^{\pi} - \frac{35}{4}(\cos 2\pi - \cos 0) = -\frac{15}{4}(\pi - 0) + \frac{15}{16}(\sin 5\pi - \sin 0) - \frac{35}{4}(1-1) = -\frac{15}{4}\pi.$$



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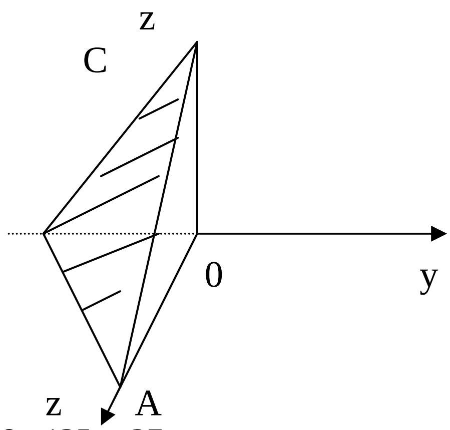
№405

$$\overline{F} = (2x+3y-3z)\overline{j};\quad 2x-3y-2z-6=0 \Rightarrow A(3;0;0); B(0;-2;0); C(0;0;3)$$

$$\overline{n} = \frac{(2;-3;2)}{\sqrt{2^2+3^2+2^2}} = \left(\frac{2}{\sqrt{17}};\frac{-3}{\sqrt{17}};\frac{2}{\sqrt{17}}\right);$$



$$1)\Phi_0 = \iint\limits_0 \overline{F}d\overline{S} = -\iint\limits_{\Delta AOS}(2x+3y-3z)\Big|_{3y=2x+2z-6} dxdz =$$

$$-\int_0^3 dx \int_0^{3-x}(4x-z-6)dt = \int_3^0\left(4xz-\frac{z^2}{2}-6z\right)\Bigg|_0^{3-x} dx =$$

$$= \int_3^0\left(12x-4x^2-\tfrac{9}{2}+3x-\frac{x^2}{2}-181+6x\right)dx =$$

$$\int_3^0\left(-\frac{9}{2}x^2+21x-\frac{45}{2}\right)dx = \left(-\frac{3}{2}x^3+\frac{21}{2}x^2-\frac{45}{2}x\right)\Bigg|_3^0 = \frac{81-198+135}{2} = \frac{27}{2}$$

2)

$$a)C = \oint\limits_\lambda \overline{F}d\overline{e} = \int\limits_{AC}\overline{F}\big|_0 d\overline{e} + \int\limits_{CB}\overline{F}d\overline{e} + \int\limits_{BA}\overline{F}d\overline{e} = \int_0^{-2}(2x+3y-3z)\Bigg|_{\substack{x=0\\ z=\frac{6+34}{2}}}^{dy} + \int_{-2}^{0}(2x+3y-3z)\Bigg|_{\substack{z=0\\ 2x=6+3y}}^{dy} =$$

$$= \int_{-2}^{0}(3y+34+6-3y+9+\tfrac{9}{2}y)dy = \int_{-2}^{0}\left(\frac{15y}{2}+15\right)dy = \left(\frac{15}{4}y^2+15y\right)\Bigg|_{-2}^{0} = -15+30=15$$

$$б)20t\overline{F} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 2x+34-3z & 0 \end{vmatrix} = 3\overline{i}+2\overline{k}$$

$$C = \iint\limits_0 20t\overline{F}d\overline{S} = \iint\limits_{\Delta ABC}\left(3\frac{2}{\sqrt{17}}+2\frac{2}{\sqrt{17}}\right)dS = \frac{10}{\sqrt{17}}\iint\limits_{\Delta ABS}dS = 5\iint\limits_{\Delta AOB}dxdy = 5S_{\Delta AOB} = \frac{5}{2}|OA||OB| = \frac{5}{2}3*2=15;$$

$$3)a)\Phi_{пир} = \iint\limits_{\Delta ABC}\overline{F}d\overline{S} + \iint\limits_{\Delta AOC}\overline{F}d\overline{S} + \iint\limits_{\Delta BOC}\overline{F}d\overline{S} + \iint\limits_{\Delta AOB}\overline{F}d\overline{S} = \frac{27}{2} + \iint\limits_{\Delta AOC}(2x+3y-3z)\Bigg|_{y=0}^{dz=dx} = \frac{27}{2}\int_0^{3-x}(2x-3z)dz =$$

$$= \frac{27}{2}+\int_0^3\left(2xz-\frac{3}{2}z^2\right)\Bigg|_0^{3-x} = \frac{27}{2}+\int_0^3\left(6x-2x^2-\frac{27}{2}+9x-\frac{3}{2}x^2\right)dx = \frac{27}{2}+\int_0^3\left(-\frac{7}{2}x^2+15x-\frac{27}{2}\right)dx =$$

$$= \frac{27}{2}+\left(-\frac{7x^3}{6}+\frac{15}{2}x^2-\frac{27x}{2}\right)\Bigg|_0^3 = \frac{27}{2}-\frac{63}{2}+\frac{135}{2}-\frac{81}{2} = \frac{18}{2} = 9$$

$$б)\Phi_{пир} = \iiint\limits_{пир} divFdV = 3\iiint\limits_{пир} dV = 3V_{пир} = \frac{1}{2}|OA|OB|OC| = \frac{1}{2}*2*3*3 = 9.$$

№415

$F = (4x - 7yz)\bar{i} + (4y - 7xz)\bar{j} + (4z - 7xy)\bar{k}$

$$rot\bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 4x - 7yz & 4y - 7xy & 4z - 7xy \end{vmatrix} = \bar{i}\left(\frac{\partial(4z-7xy)}{\partial x} - \frac{\partial(4y-7xz}{\partial z}\right) - \bar{j}\left(\frac{\partial(4z-7xy)}{\partial x} - \frac{\partial(4x-7xz)}{\partial z}\right) +$$

$$+\bar{k}\left(\frac{\partial(4y-7xy)}{\partial x} - \frac{\partial(4x-7yz)}{\partial y}\right) = \bar{i}(-7x+7x) - \bar{j}(-7y+7y) + \bar{k}(-7z+7z) = 0 \Rightarrow$$ поле является потенциальным. Найдём потенциал поля:

$$u(x,y,z) = \int_0^x P(x,y_0,z_0)dx + \int_0^y Q(x,y,z_0)dy + \int_0^z R(x,y,z)dz + C = \int_0^x (4x)dx + \int_0^y 4ydy + \int_0^z (4z-7xy)dz =$$

$$= 2x^2 + 2y^2 + 2z^2 - 7xyz + C$$

$$divF = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = (4x-7yz)'_x + (4y-7xz)'_y + (4z-7xy)'_z = 4+4+4+4 = 16 \neq 0$$

Поле не является соленоидальным.

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