№428

$$\sum_{n=1}^{\infty}\frac{n^2}{(3n)!}$$

Исследовать сходимость.

Решение:

$$U_n=\frac{n^2}{(3n)!}$$

$$U_{n+1}=\frac{(n+1)^2}{(3(n+1))!}=\frac{(n+1)^2}{(3n+3)(3n+2)(3n+1)(3n)!}$$

Применим признак Даламбера:

$$D=\lim_{n\to\infty}\frac{(n+1)^2*3n!}{(3n+3)(3n+2)(3n+1)!n^2}=\lim_{n\to\infty}\left(\left(\frac{n+1}{n}\right)^2*\frac{1}{(3n+3)(3n+2)(3n+1)}\right)=$$

$$=\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^2\lim_{n\to\infty}*\frac{1}{(3n+3)(3n+2)(3n+1)}=1*0=0$$

ряд $\sum_{n=1}^{\infty}\frac{n^2}{(3n)!}$ - сходится.

№438

$$\sum_{n=1}^{\infty}\frac{(n+1)(x^n)}{3^n(n+2)}$$

Интервал сходимости.

Решение:

$$a_n=\frac{n+1}{3^n(n+2)}$$

$$a_{n+1}=\frac{n+2}{3^{n+1}(n+3)}$$

$$R=\lim_{n\to\infty}\frac{(n+1)3^{n+1}(n+3)}{3^n(n+2)(n+2)}=3\lim_{n\to\infty}\frac{(n+1)(n+3)}{(n+2)^2}=3\lim_{n\to\infty}\frac{n^2+4n+3}{n^2+4n+4}=3\lim_{n\to\infty}\frac{1+\frac{4}{n}+\frac{3}{n^2}}{1+\frac{4}{n}+\frac{4}{n^2}}=3$$

$R=3$ Интервал сходимости:]-3;3[.

№448

$f(x)=\sin x^2,\qquad b=1$

$\int_0^b f(x)dx$ - ?

Решение:

Определим количество необходимых членов ряда:

$\sin x^2 = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \ldots$

$$\int_0^1 \sin x^2 dx \int_0^1 \left(x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \ldots \right) dx = \frac{x^3}{3} - \frac{x^7}{6*7} + \frac{x^{11}}{20*11} - \Big|_0^1 = \frac{1}{3}(1^3 - 0^3) - \frac{1}{42}(1^7 - 0^7) +$$

$$+ \frac{1}{1320}(1^{11} - 0^{11}) + 0{,}333 - 0{,}0238 + 0{,}00076 - \ldots \approx 0{,}333 - 0{,}024 \approx 0{,}309.$$

http://kvadromir.com/arutunov_sbornik_10.html — решебник Арутюнова Ю.С. Контрольная работа 10. Вариант 8. Номера 428, 438, 448, 458, 468

№458

$y' = \sin x + 0{,}5y^2 \qquad y(0)=1$

$$y = y(x) = y(0) + \frac{y'(0)}{1!}x + \frac{y''(0)}{2!}x^2 + \frac{y'''(0)}{3!}x^3 + \ldots + \frac{y^n}{n!}x^n$$

$$y'(0) = \sin 0 + 0{,}5*1^2 = \frac{1}{2}$$

$$y''(x) = \cos x + y*y'$$

$$y''(0) = \cos 0 + 1*\frac{1}{2} = 1 + \frac{1}{2} = \frac{3}{2}$$

$$y'''(x) = -\sin x + y'y' + yy''$$

$$y'''(0) = -\sin 0 + \frac{1}{2}*\frac{1}{2} + 1*\frac{3}{2} = \frac{1}{4} + \frac{3}{2} = \frac{1+6}{4} = \frac{7}{4}$$

$$y = y(x) = 1 + \frac{1}{2}x + \frac{3}{4}x^2 + \frac{7}{24}x^3 + \ldots \quad .$$

http://kvadromir.com/arutunov_sbornik_10.html — решебник Арутюнова Ю.С. Контрольная работа 10. Вариант 8. Номера 428, 438, 448, 458, 468

№468

$$f(x) = x - 1 \qquad (-1;1) \qquad l = 1$$

$$f(x) = \frac{a_0}{2} + \sum a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}$$

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)dx = \frac{1}{1}\int_{-1}^{1}(x-1)dx = \left(\frac{x^2}{2} - x\right)\Bigg|_{-1}^{1} = \frac{1^2 - (-1)^2}{2} - (1-(-1)) = \frac{1-1}{2} - (1+1) = -2;$$

$$a_n \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{n\pi x}{l}dx = \frac{1}{1}\int_{-1}^{1}\cos\frac{n\pi x}{1}dx = \int_{-1}^{1}(x-1)\cos n\pi x dx =$$

$$\left|\begin{array}{l} u = x - 1 \qquad dv = \cos n\pi x dx \\ du = (x-1)'dx = dx \\ v = \int \cos n\pi x dx = \frac{1}{n\pi}\sin n\pi x \\ \int udv = uv - \int vdu \end{array}\right.$$

$$= (x-1)\frac{1}{n\pi}\sin n\pi x\Bigg|_{-1}^{1} - \int_{-1}^{1}\frac{1}{n\pi}\sin n\pi x dx = (1-1)\frac{1}{n\pi}\sin n\pi - (-1-1)\frac{1}{n\pi}\sin n\pi(-1) + \frac{1}{n^2\pi^2}\cos n\pi x\Bigg|_{-1}^{1} =$$

$$= 0 + \frac{2}{n\pi} * 0 + \frac{1}{n^2\pi^2}(\cos n\pi - \cos(-n\pi)) = 0 + \frac{1}{n^2\pi^2}(\cos n\pi - \cos n\pi) = 0;$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi x}{l}dx = \frac{1}{1}\int_{-1}^{1}(x-1)\sin n\pi x dx =$$

$$\left|\begin{array}{l} u = (x - 1) \qquad dv = \sin n\pi x dx \\ du = (x-1)'dx = dx \\ v = \int \sin n\pi x dx = -\frac{1}{n\pi}\cos n\pi x \\ \int udv = uv - \int vdu \end{array}\right.$$

$$= (x-1)\left(-\frac{1}{n\pi}\cos n\pi x\right)\Bigg|_{-1}^{1} - \int_{-1}^{1}\left(-\frac{1}{n\pi}\cos n\pi x\right)dx = (1-1)\left(-\frac{1}{n\pi}\cos n\pi 1\right) - (-1-1)\left(-\frac{1}{n\pi}\cos n\pi(-1)\right) +$$

$$+ \frac{1}{n^2\pi^2}\sin n\pi x\Bigg|_{-1}^{1} = 0 - \frac{2}{n\pi}(-1)^n + \frac{1}{n^2\pi^2}(\sin n\pi - \sin(-n\pi)) = \frac{2}{n\pi}(-1)^{n+1}$$

$$f(x) = -\frac{2}{2} + \sum\left(0\cos n\pi x + \frac{2}{n\pi}(-1)^{n+1}\sin n\pi x\right) = -1 + \sum\frac{1}{n\pi}(-1)^{n+1}\sin n\pi x.$$